A Corollary to the Division Algorithm (New)

By

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The following result computes quotients and remainders for a negative integer dividend and a positive integer divisor whose existence is guaranteed by the division algorithm. We use the results that are done for the negative of that negative integer which is a positive integer and then reverse the whole process. But when we reverse the process we have to address the would be remainder to be at least non negative and at most $d-1$ to be in the frame of the algorithm.

Here is the reslut:

Theorem. Let $n \in \mathbb{Z}^-$ and $d \in \mathbb{Z}^+$. Then $\exists!$ positive integers q, r with $0 \leq r < d$ such that $n = -(q+1)\,d + (d-r)$. That is;

$$ndivd = -(q+1) \quad \text{and} \quad n \bmod d = (d-r)\,.$$

Proof. $n \in \mathbb{Z}^- \Rightarrow -n \in \mathbb{Z}^+$. Then as d is a positive integer, by the division algorithm $\exists! q, r \in \mathbb{Z}^+$ such that $0 \leq r < d$ and

$$-n = qd + r \Rightarrow \frac{-n}{d} = q + \frac{r}{d}.$$

Then rewriting n as $-(-n)$ we have :

$$\frac{n}{d} = -\left(\frac{-n}{d}\right) = -\left(q + \frac{r}{d}\right) = -q - \frac{r}{d}$$

Looking at the remainder here at the last expression, we change that to a positive remainder by adding 1 since $\frac{r}{d} < 1$.

That is

$$\frac{n}{d} = -q - \frac{r}{d} = -q + (-\frac{r}{d} + 1) - 1 = -(q+1) + \frac{d-r}{d}$$

In the last expression we see that the new expression for the remainder is $(d-r)$ which again satisfies the inequality: $0 \leq d - r < d$.

Thus rewriting the algebraic expression $\frac{n}{d} = -(q+1) + \frac{d-r}{d}$ we have :

$$n = -(q+1)\,d + (d-r)\,.$$

To write it differently:

$$n = -(-ndivd + 1)\,d + (d - (-n \bmod d))$$