

Problem 7. Let  $f \in C^1(\mathbb{R}, \mathbb{R})$  with properties :

(i)  $f(x+y) = f(x)f(y)$

(ii)  $f(x-y)f(y) = f(x)$ .

Then show that the function indeed is given by:

$f(x) = ce^{\alpha x}$  where both  $c, \alpha$  are some real constants with  $c$ -non-zero and

$$\alpha = \lim_{h \rightarrow 0} \left( \frac{f(h)-1}{h} \right)$$