

Problem 9. Let  $H$  be the Heavyside function and  $v$  be an infinitely many times differentiable function with compact support over  $\mathbb{R}$  ( i.e.  $\lim_{|x| \rightarrow \infty} v(x) = 0$  ).

Show that indeed  $H$  is infinitely many times differentiable over  $\mathbb{R}$  in a distribution sense but not in a Riemann sense and

$$\sum_{k=1}^{\infty} \int_{\mathbb{R}} \frac{d^k H(x)}{dx^k} \cdot \frac{v(x)}{(k-1)!} dx = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{d^{k-1} v(0)}{(k-1)!}$$