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Complete Function Systems and Decomposition Results Arising in Clifford Analysis

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Abstract. For $\Omega$ a sufficiently smooth unbounded domain in $\mathbb{R}^n$ we develop a decomposition result for the Sobolev space $W_{\mathcal{C}_{\partial,\Omega}}^{p,l-1}(\Omega)$. We also use modified Cauchy-Green type kernels to construct Clifford analytic-complete function systems in the generalized Bergman space $B_{\mathcal{C}_{\partial,\Omega}}^{p,l}(\Omega) := \ker D_l(\Omega) \cap W_{\mathcal{C}_{\partial,\Omega}}^{p,l-1}(\Omega)$, where $D_l$ is the $l$-th iterate of the Dirac operator, $l$ is a positive integer less than $n$ and $n/(n-l+1) < p < \infty$. The modified Cauchy-Green kernels ensure that $p$ lies in this range. Without the modification of the kernels one is restricted to a smaller range. These functions are used to approximate solutions of the equation $\Delta^k u = 0$ with some boundary conditions and with $2k < n$. Some similar results are presented for sufficiently smooth unbounded domains lying in hyperbolas.

Keywords. Clifford Analysis, complete function systems, Bergman spaces, decomposition spaces, elliptic boundary value problems, Dirac operators and hyperbolas.

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