Colleagues,

I will post my second communication.

Let us define another differential operator of infinite terms as :

$e^{-D} := \sum_{j=0}^{\infty} \frac{(-1)^j D^{(j)}}{j!}$, when $j = 0$, we have the identity operator,

and $D := \frac{d}{dx}$

Then as in my first communication post, we can question the following:

$(\forall \psi \epsilon C^{\infty}(I, \mathbb{R})) \Lambda (\forall x \epsilon I)$, what will be $e^{-D}(\psi(x)) = \sum_{j=0}^{\infty} \frac{(-1)^j D^{(j)} \psi(x)}{j!}$?

Consider the following example:

Example 1: Take $\psi(x) = e^x$ the usual natural exponential function.

<u>Claim</u>: $e^{-D}(\psi(x)) = \psi(x-1)$.

Indeed,

$$\begin{aligned} e^{-D}(\psi(x)) &= \sum_{j=0}^{\infty} \frac{(-1)^j D^{(j)} \psi(x)}{j!} \\ &= \sum_{j=0}^{\infty} \frac{(-1)^j D^{(j)} (e^x)}{j!} \\ &= \sum_{j=0}^{\infty} \frac{(-1)^j e^x}{j!} \\ &= e^x \underbrace{\sum_{j=0}^{\infty} \frac{(-1)^j}{j!}}_{\substack{\| \\ e^{-1}}} \\ &= e^{x-1} = \psi(x-1) \end{aligned}$$

$\therefore \quad e^{-D}\psi(x) = \psi(x-1)$ ... which is a right translation of ψ by a unit.

One can extend this result further and write a corollary as :

Corollary: $(\forall k \epsilon \mathbb{N})$: $\left(e^{-D}\right)^k \psi(x) = \psi(x-k)$-right translate of ψ by k-units.

Example 2. Let $\phi(x) = x^3 + x^2 + x + 1$.Then

$$\begin{aligned} e^{-D}\phi(x) &= \sum_{j=0}^{\infty} \frac{(-1)^j D^{(j)} \phi(x)}{j!} \\ &= \sum_{j=0}^{\infty} \frac{(-1)^j D^{(j)} \left(x^3+x^2+x+1\right)}{j!} \\ &= x^3 - 2x^2 + 2x \end{aligned}$$

But the expression we have at the end is precisely $\phi(x-1)$.

That is once again we have a similar result :

$$e^{-D}\phi(x) = \phi(x-1)$$

Corollary: $\forall\ p(x)\epsilon\wp(x)$, $e^{-D}p(x) = p(x-1)$

Conjecture : $\forall\psi\epsilon C^{\infty}(I,\mathbb{R})$, $e^{-D}\psi(x) = \psi(x-1)$

Corollary: $(\forall k\epsilon\mathbb{N})\,(\forall\psi\epsilon C^{\infty}(I,\mathbb{R})), e^{-kD}\psi(x) = \psi(x-k)$

Further communications will be posted on operators defined from combinations of both.