Problem 8. Suppose $\psi \in C^1 (\mathbb{R}_{>0}, \mathbb{R}) \ni$ the following hold:

(i) $\psi(xy) = \psi(x) + \psi(y)$

(ii) $\psi(x) + \psi\left(y^{-1}\right) = \psi(x) - \psi(y)$

(iii) $\psi(1) = 0$

Then show that the function is given by $\psi(x) = \psi(e) \ln x$