The non- Mathematical World of Cartoons

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There are no 2-D ears and 2-D spherical sound sources

How do cartoons that we see in television communicate? Do flat landers have audio communication abilities? These questions can be generalized in one basic quest of natural phenomenon: Do spherical sound waves exist in 2-D spaces ? To answer these questions, we search for a radially symmetric equation of motion in 2-D which is similar to a flat land that will solve the radially symmetric 2-D wave equation given by the partial differential equation:

$$\Psi_{tt}(r,t) = \frac{\delta}{r^2}\left(r\Psi_r(r,t)\right)_r$$

where $()_r := \frac{\partial}{\partial r}()$ and $()_t := \frac{\partial}{\partial t}()$ and δ is some positive constant.

Here, the function $\Psi(r,t)$ is the solution to the equation of motion that we are searching for, that satisfies the PDE given above. If we can find such integrals, for the radially symmetric wave equation described by the PDE, then indeed we say that cartoons do make audio communications in the way we see from films prepared by Hollywood and Disney channels.

But actually that is not the case. We will see that the above partial differential equation indeed has no radially symmetric integral surface.

Let us assume that the wave equation has a solution $\Psi(r,t)$ which is given by a special form : $\Psi(r,t) = \psi(r)\,\phi(r-vt)$ where, $\psi(r)$ is a radial function that measures the radial distance of a wave front from the source and $\phi(r-vt)$ is a velocity function of wave fronts at time t. The splitting of the solution function as the product of the two functions is due to the spherical method of separation.

Putting the given function $\Psi(r,t) = \psi(r)\,\phi(r-vt)$ in to the partial differential equation, we get the following equation:

$$\psi(r)\,v^2\phi''(r-vt) = \frac{k}{r^2}\left(\left[\psi'(r) + r\psi''(r)\right]\phi(r-vt) + \left[\psi(r) + 2\psi'(r)\right]\phi'(r-vt) + \left[r\psi(r)\,\phi''(r)\right]\right)$$

where k is some real constant. Then equating coefficients of ϕ, ϕ' and ϕ'' on both sides of the above equation we get a set of equations given by:

$$\begin{cases} \psi'(r) + r\psi''(r) = 0 \\ \psi(r) + 2r\psi'(r) = 0 \end{cases}$$

Then we get the following equivalent set of differential equations :

$$\begin{cases} \psi'(r) = \frac{\alpha}{r} \\ \left(\psi'(r)\right)^2 = \frac{\beta}{r} \end{cases}$$

where α and β are some non negative constants. The last system of equations is solvable only when $\alpha = \beta = 0$.

Therefore this result forces the function $\psi(r)$ be a constant function of the radius. But, the value of the radial function at $r = 0$ is zero, that is $\psi(r)\mid_{(r=0)} = 0$. Then as ψ is constant, we get that $\psi \equiv 0$.

Therefore, the solution function is given by

$$\Psi(r,t) = \psi(r)\,\phi(r - vt) = 0$$

for all $r \geq 0$ and for all $t \geq 0$. That is $\psi \equiv 0$ in the domain of flat landers.

The conclusion is that, there was not a sound wave that comes out of the planar mouth of a flat lander and goes to the planar ears of the other flat lander or cartoon character.

Hence, cartoons of Hollywood or Disney do not communicate in audio contrary to what we see in television where they infact communicate as we do.

To put it differently, our ears do not recieve planar crossections of audible sound waves. Is it not interesting?