Colleagues, recently we have a new group " International Professors " added to our groups. It is therefore good to create a new category in order we share insights,

new methods, interesting class encounters and new concepts introduced when teaching undergraduate mathematics. This will create a plat form to share how

curriculums are apart or close on global setting and might give a hint to education policy makers what they have to expect from undergraduate mathematics curriculums

in order to go at par with international standards.

I will present my first communication.

It is on enlarging the usual differential operator $D := \frac{d}{dx}$ in variable x to something else. We know that the usual differentiation makes functions to loose their smoothness or

regularity by a degree (if they are not infinitely many times continuously differentiable)

The types of questions I have, can therefore be given as extra exercises or new insights to students who take calculus on sequences, series and convergence,

to engage them to think more about, not only the single calculus operations but, combined of them and thereby do algebraic computations at the same time.

Let us define a new differential operator of infinite terms as :

$\sum_{j=0}^{\infty} \frac{D^{(j)}}{j!} =: e^D$, for $j = 0$, we have the identity operator.

Then for a real valued $C^{\infty}-$ function defined on some non-degenerate open interval I (or $\mathbb{R}$-for that matter)

we can question the following:

what will be the action of e^D on such functions.

Thus if $\psi \epsilon C^{\infty}(I, \mathbb{R})$, what will be $\sum_{j=0}^{\infty} \frac{D^{(j)}\psi(x)}{j!}$?

The very immediate question will be the question of summability of the series indicated?

But we take cases in which that condition works:

Example 1: Take $\psi(x) = e^x$ the usual natural exponential function.

We see that $e^D(e^x)$ converges to the sum : $e\psi(x) = \psi(x+1)$.

Indeed,

$$\sum_{j=0}^{\infty} \frac{D^{(j)}\psi(x)}{j!} = \sum_{j=0}^{\infty} \frac{D^{(j)}(e^x)}{j!}$$

$$= \sum_{j=0}^{\infty} \frac{e^x}{j!} = e^x \underbrace{\sum_{j=0}^{\infty} \frac{1}{j!}}_{\substack{\| \\ e}}$$

$$= e^{x+1} = \psi(x+1)$$

$\therefore \quad e^D \psi(x) = \psi(x+1)$-which is a left translation of ψ by a unit.

One can extend this result further and write a corollary as :

Corollary: $\left(e^D\right)^k \psi(x) = \psi(x+k)$-left translate of ψ by k-units.

Example 2. Let $\phi(x) = x^3 + x^2 + x + 1$.Then

$$e^D \phi(x) = \sum_{j=0}^{\infty} \frac{D^{(j)}\phi(x)}{j!}$$

$$= \sum_{j=0}^{\infty} \frac{D^{(j)}\left(x^3+x^2+x+1\right)}{j!}$$

$$= x^3 + 4x^2 + 6x + 4$$

But the expression we have at the end is $\phi(x+1)$.

Therefore once again we have :

$$e^D \phi(x) = \phi(x+1)$$

Claim: For a polynomial function $p(x)$, $e^D p(x) = p(x+1)$

Conjecture: $\forall \psi \epsilon C^\infty(I, \mathbb{R})$, $e^D \psi(x) = \psi(x+1)$

We can also define a similar operator that results in right translations of $C^\infty-$functions by counts of units as:

$$e^{-D} := \sum_{j=0}^{\infty} \frac{(-1)^j D^{(j)}}{j!}$$

Further communications will be posted on the last operator and combinations of both.