I will post my third communication.

Let us define another differential operator of infinite terms as :

$$\mathcal{L}^{\infty,odd} := \sum_{k=0}^{\infty} \frac{D^{(2k+1)}}{(2k+1)!}$$

where $D := \frac{d}{dx}$

Then as in my first and second communications, we can question the following:

$(\forall \psi \epsilon C^{\infty}(I, \mathbb{R})) \Lambda (\forall x \epsilon I)$, what will be

$$\mathcal{L}^{\infty,odd}(\psi(x)) = \sum_{k=0}^{\infty} \frac{D^{(2k+1)}\psi(x)}{(2k+1)!}?$$

Consider the following example:

Example 1: Take $\psi(x) = e^x$ the usual natural exponential function.

Claim: $\mathcal{L}^{\infty,odd}(\psi(x)) = \psi(x+1) - \psi(x-1)$.

Indeed,

$$\begin{aligned} \mathcal{L}^{\infty,odd}(\psi(x)) &= \sum_{k=0}^{\infty} \tfrac{D^{(2k+1)}\psi(x)}{(2k+1)!} \\ &= \sum_{k=0}^{\infty} \tfrac{D^{(2k+1)}(e^x)}{(2k+1)!} \\ &= e^{x+1} - e^{x-1} \\ &= \psi(x+1) - \psi(x-1) \end{aligned}$$

Example 2. Let $\phi(x) = x^3$.Then

$$\begin{aligned} \mathcal{L}^{\infty,odd}\phi(x) &= \sum_{k=0}^{\infty} \tfrac{D^{(2k+1)}\phi(x)}{(2k+1)!} \\ &= \sum_{k=0}^{\infty} \tfrac{D^{(2k+1)}(x^3)}{(2k+1)!} \\ &= 6x^2 + 2 \end{aligned}$$

But the expression we have at the end is precisely: $\phi(x+1) - \phi(x-1)$

That is :

$$\mathcal{L}^{\infty,odd}\phi(x)=\phi(x+1)-\phi\left(x-1\right)$$

In a similar argument, we can show that :

P**roposition**: $\forall\ p(x)\epsilon\wp\left(x\right)$, $\mathcal{L}^{\infty,odd}p(x)=p(x+1)-p(x-1)$, where $\wp\left(x\right)$ is the set of all polynomial functions in x

Conjecture : $\forall\psi\epsilon C^{\infty}\left(I,\mathbb{R}\right),\,\mathcal{L}^{\infty,odd}\psi\left(x\right)=\psi\left(x+1\right)-\psi\left(x-1\right)$

Again for the differential operator:

$$\mathcal{L}_{k}^{\infty,odd}:=\sum_{j=0}^{\infty}\frac{k^{(2j+1)}D^{(2j+1)}}{(2j+1)!}$$

where D is the differential operator above we have:

Conjecture: $\left(\forall k\epsilon\mathbb{N}\right)\left(\forall\psi\epsilon C^{\infty}\left(I,\mathbb{R}\right)\right),\mathcal{L}_{k}^{\infty,odd}\psi\left(x\right)=\psi\left(x+k\right)-\psi\left(x-k\right)$

N.B.:The fourth communication will be posted shortly.